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DOI: https://doi.org/10.34220/issn.2222-7962/2023.3/4
УДК 630*561

Simple asymmetry indicators tested in natural even-aged Scots pine (*Pinus sylvestris* L.) stands in Bulgaria

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Abstract

Based on 101 sample plots laid in Scots pine forest dendrocenoses aged 72 as an average with fulness around and over 0.7 has the structure in thickness, more exactly – the asymmetry of the distribution of tree numbers according to thickness, been investigated. Based on a retrospective analysis theoretically proved new indices have been introduced for finding and investigating this asymmetry, as well as a new indicator of asymmetry. The results obtained show that: a). The indices of asymmetry and the indicator of asymmetry, which proceeds from the indices, are in a close correlation (R^2 = 0.93; 0.94) with the coefficients of asymmetry and are suitable for generalised comparative studies. b). The currently used generalised indicator of asymmetry – the zero natural indicator has a weaker correlation with the coefficient of asymmetry as compared with the correlation between the index of asymmetry and the indicator of asymmetry. c). The comparison between the indices of asymmetry for thickness structure and the indices of steepness for height structure has revealed a common trend as to inversely proportional change in these two kinds of indices and their correlation is R = 0.26. The general conclusion is that the indices and indicators of asymmetry are completely suitable and effective for generalised comparative studies such as, for example, the comparison between the structures in thickness and height of forest dendrocenoses.

Keywords: *forest biometry, diametric distributions, Pinus sylvestris* L., *skewness, height curves, steepness.*

Funding: this research received no external funding.
Acknowledgments: the author thanks the reviewers for their contribution to the peer review of this paper.
Conflict of interest: the author declares no conflict of interest.

For citation: Petrin R. D. (2023). Simple asymmetry indicators tested in natural even-aged Scots pine (*Pinus sylvestris* L.) stands in Bulgaria. *Lesotekhnicheskii zhurnal* [Forestry Engineering journal], Vol. 13, No. 3 (51), pp. 41-55 (in Russian). DOI: https://doi.org/10.34220/issn.2222-7962/2023.3/4.

Received 18.05.2023. ***Revised*** 03.11.2023. ***Accepted*** 13.11.2023. ***Published online*** 30.11.2023.

Оригинальная статья

Простые показатели асимметрии, использованные при исследовании естественных одновозрастных насаждений сосны обыкновенной (*Pinus sylvestris* L.) в Болгарии

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На основе 101 пробной площади в древостоях сосны обыкновенной (*Pinus sylvestris L.*) с возрастным диапазоном от 25 до 130 лет исследована асимметрия распределений деревьев по диаметру. На основании ретроспективного анализа предложены простые в использовании и подходящие для научных исследований строения лесных насаждений показатели – индекс асимметрии и показатель асимметрии. Предложенные показатели находятся в тесной корреляции друг с другом и могут считаться эквивалентными. Известный в более ранних публикациях специализированный показатель асимметрии – так называемый нулевой натуральный показатель, имеет с ними более слабую корреляцию. Предложенный индекс асимметрии диаметрального распределения обнаруживает обратную корреляцию с индексом крутизны кривых высот. Показатели могут применяться и пригодны для сравнительных исследований лесных дендроценозов по диаметру и высоте.

Ключевые слова: *лесная биометрия, диаметральные распределения, Pinus sylvestris* L.*, кривые высот, асимметрия, крутизна.*

Финансирование: исследование не получало внешнего финансирования.

Благодарности: автор выражает благодарность рецензентам за вклад в экспертную оценку статьи.

Конфликт интересов: автор заявил об отсутствии конфликта интересов.

Для цитирования: Петрин, Р. Д. Простые показатели асимметрии, использованные при исследовании естественных одновозрастных насаждений сосны обыкновенной (*Pinus sylvestris* L.) в Болгарии / Р. Д. Петрин // Лесотехнический журнал. – 2023. – Т. 13. – № 3 (51). – С. 41–55. – *Библиогр.: с. 51–55 (35 назв.).* – DOI: https://doi.org/10.34220/issn.2222-7962/2023.3/4.

Поступила 18.05.2023. ***Пересмотрена*** 03.11.2023. ***Принята*** 13.11.2023. ***Опубликована онлайн*** 30.11.2023.

Introduction

An important problem with measuring the volumes of forest-stand trees and with the formation of assortments from these trees is the one with the shapes of the variation curves of the distribution of tree numbers according to thickness levels and, more exactly, where the maximum of these numbers is with respect to the average diameter (d_{av}) – on the left or on the right, as the side on which the summed numbers of trees are more is of decisive importance. A variation curve with a right-hand-side asymmetry signifies, for a particular stand or an aggregate of stands, a larger amount of bigger-sized timber. A left-hand-side asymmetry is available when numbers of thinner trees dominate, i.e. when the maximum of these numbers is situated on the left of d_{av}, or around it when the distribution curve passes at a greater height. It is just the opposite with the right-hand-side asymmetry: the distribution curve's maximum or passing at a greater height is on the right of $d_{av.}$ And when the maximum is around the average diameter, the distribution curve, also called thickness-structure curve, is of a symmetric (or normal) type. The maximum with the symmetric type of distribution is around the average diameter (d_{av}), and it is necessary to have in mind the established scientific fact (Tretyakov, 1927), that the average diameter divides the total number of trees in even-aged dendrocoenoses into two unequal proportions in the following ratio: about 60% thinner and 40% thicker. This is why, for higher precision of the investigation, the asymmetry found with respect to the average diameter, as measured through forest-mensuration methods, has been preferred to the statistical asymmetry determined with respect to the maximum (i.e. moda) of the parabolic curve.

The availability of different types of thickness structure, or types of asymmetries of distribution, is a fact known since long ago as found for many tree species. Tretyakov (1927), based on data of his own and data taken from Weise, Kunze and others, carried out a detailed study of the regularities in the structure and variability of some dendrobiometric indicators found in even-aged, pure and uneven-aged stands, with mixed composition and complex management. He found out that the forest structure always had a constant nature regardless of spacing index, age, tree species, growth conditions and whether the stands were normal or complex, mixed ones. This gave him the reason to formulate the Law of Uniformity of Stand Structures.

S. Nedyalkov (1955) found out for high-stemmed beech stands a difference in the shapes of the variation curves of distribution depending on the thinnings carried out in the stands, as the total percentage of the number of trees of diameters close to the average one was by 3-4% larger for the stands where no thinnings had been carried out. Similar differences in the total percentage of trees thinner than the average diameter were found out for mixed beech-and-Scots-pine stands by Krustanov (1968) depending mainly on age. E. P. Dimitrov (1978) found out for mature beech stands the availability of three typical distributions: symmetric, or normal, right-hand-side asymmetric and left-hand-side asymmetric. E. T. Dimitrov (2003), while investigating different functions of tree-number distribution according to levels of thickness and height, found out the availability of three distributions typical of Scots pine, Norway spruce and silver fir: symmetric, or normal, right-hand-side asymmetric, or positive, and left-hand-side asymmetric, or negative*. (* In some of the investigations cited, it is talked «*about a statistical asymmetry of the very curve with respect to its maximum, not about an asymmetry with respect to the average diameter, i.e. with respect to a fixed point on the abscissa. Most often, a left-hand-side negative statistical asymmetry corresponds to the right-hand-side asymmetry with respect to the average diameter, and a right-hand-side positive statistical asymmetry corresponds to the left-hand-side asymmetry with respect to* d_{av}»)

R. Petrin, independently as well as in co-authorship, has also found out the availability of «*three different types of asymmetry with respect to* d_{av}*: left-hand-side, right-hand-side and symmetric, or normal, type*» for seed-tree common beech (2013, 2014), for spruce and fir (2015), for coppice dendrocoenoses of Hungarian oak, durmast and Turkey oak (2016), for Scots pine and Scots pine crops (2018), and for seed-tree dendrocoenoses of Hungarian oak, durmast and Turkey oak (2019).

In pieces of Western literature, the steepness of growth curves has been called rate of rate or trend of growth (Mario Trouillier et al., 2020). According to a similar logic in our investigation has the steepness of the height-structure curves been found out by means of the indices of steepness I_{st} (Petrin, 2021). Diego Rodriguez de Prado, Jose Riorfio et al. (2022) have investigated the height structures of pure and mixed coniferous and deciduous stands. They have found out that pure coniferous stands reach greater average heights than mixed ones, whereas with deciduous stands, on the contrary, mixed ones grow better in height.

Kyle W. Tomlinson et al. (2014) have found out that the growth rate of coniferous trees is higher than that of deciduous ones. Louisa Timinska-Chabanska et al. (2020) have found out for the main forest-forming (pioneer) forest tree species that the curves of the site-index (stand-quality-level) classes of the young and old stands divert to the greatest extent from the average positions of these. Facundo J. Odie, Cecilia Callas et al. (2022) have found out about Chilean cedar (*Austrocedrus chilensis*) that it grows better on moist and cool sites where there is more carbon in the soil and lower acidity.

Purpose of Investigation

The objectives of the present work are:

1. To propose an asymmetry index for diametrical distributions in forestry, as well as the asymmetry rate related with him.

2. To find out the typical values of the index, e.g. in natural Scots pine stands.

3. To apply the index to study the relationship between the asymmetry of the diametrical distributions and the steepness of the height curves.

Objects and Methods

A total of 110 sample plots in natural Scots pine stands aged from 25 to 130 years (72 as an average) with fulness around and over 0.7 were used for obtaining the data on thickness structure, and 27 – for the data on height structure. As usual, "forest stand structure by thickness" means the diametrical distribution of tree number, and "forest stand structure by height" – the height curve.

All curves we consider were standardized by conversion into **natural levels of thickness** (NLT), i.e. by dividing the diameters by the mean diameter of the forest stand,

$$x = \frac{d}{d_{\text{mean}}} = \text{NLT}. \quad (1)$$

NLT values were rounded to the nearest 0.1. The **central level of thickness** corresponds to the mean diameter d_{mean} . Thus, for the central level of thickness $x = 1$. NLT are usually said to range from 0.5 to 1.7. NLT are usually said to range from 0.5 to 1.7. The data often violate this rule. In our data the NLT ranged from 0.5 to 1.5, i.e. outside this range the registered tree numbers were 0 or maximum 3% of total sum.

We shell term **asymmetry coefficient** the quantity

$$C = \frac{1}{N}\left(\frac{N_1}{2} + \sum_{x<1} N_x\right), \quad (2)$$

where

N_x is the number of trees in the NLT x;

$N = \sum N_x$ is the total number of trees in the stand;

N_1 is the stem number of the central NLT.

Apparently, C is simply the empiric probability that a randomly selected diameter d is smaller than the mean diameter, $C = P[d \leq d_{\text{mean}}]$. The $N_1/2$ term reflects the fact that the central degree of thickness contains an equal number of diameters larger and smaller than d_{mean}.

The interval assumed for the forest-mensuration symmetric (normal) type of distribution is C from 0.57 up to 0.61 (Tyurin, 1938), C bigger than 0.61 corresponds to a left-hand-side asymmetry, and C smaller than 0.57 – to a right-hand-side asymmetry.

We shell term **asymmetry index** the quantity

$$I = \frac{N}{\sum_{x\leq 1} N_x}. \quad (3)$$

As the Tyurin distribution has an I-value about 1,5, approximately normal distributions have I of about 1,5, lower I indicates left skew, higher – right skew.

Apparently, I is approximately reciprocal to C. It includes a simplification, since it replaces the sum $N_1/2 + \sum_{x<1} N_x$ with $\sum_{x\leq 1} N_x$. Thus

$$I \approx \frac{1}{C}, \quad I \leq \frac{1}{C}. \quad (4)$$

Finally, we shall term **asymmetry rate** the relative index

$$A = \frac{I}{I_{\text{normal}}}, \quad (5)$$

where $I_{\text{normal}} \approx 1.5$ is the asymmetry index of the Tyurin distribution. Obviously, approximately normal distributions have $A = 1$, lower A-values indicate left skew, higher – right skew.

To assess asymmetry, the so called **zero natural indicator** (ZNI) has been used by many Bulgarian publications (Douhovnikov and Mihov, 1983; Petrin, 2019). ZNI is a method used with varying success for a variety of tasks. Its main idea is to model with a linear equation the deviation of the experimental curves from the theoretical ones. When applied to standardized diametrical distributions, the method models the experimental distribution N_x with the equation

$$N_x = [b + (1 - b)x]\, n_x, \quad (6)$$

where n_x is Tyurin's, or mean for the aggregate, distribution, distribution, and b is a coefficient established by regression (and called ZNI). Distributions close to that of Turin have ZNI=1. Lower ZNI-values indicate right skew, higher – left skew.

To facilitate research, Normal curves of the distribution q_{xn}, i.e. the curves of the normal numbers were calculated, according to the formula:

$q_x = N_x/N_m$. (7), where

N_x is the numbers of trees in the individual degrees of thickness **x**,

N_m - the value of N in the central degree of thickness **m**

The normal sum point's curves q_{x_sum}, are calculated by means of the formula:

$q_{x\_sum} = N_{x\_sum}/N_{m\_sum}$ (8),

where N_{x_sum} are the values of N_{sum} for the particular levels of thickness **x**,
N_{m_sum} – the value of N_{x_sum} for the central level of thickness **m** (1.0)
The index of asymmetry is the rightmost (**Ω**) value of the curve q_{x_sum}, i.e. $\mathbf{I = q_{\Omega\ sum}}$.

The structure in height (H) is in its turn characterised by ***the steepness of the curve of heights***, which shows the increase in the growth in height with the increase in thickness; it is assessed by means of the ***index of steepness*** (Petrin, 2021). The indices of **steepness** with height structure H (I_{st}) are relative numbers calculated by means of the formula:

$$\mathrm{Ist} = \frac{H_{\Omega}}{H_1}, \quad (9)$$

where H_{Ω} is the height of the trees in the high levels of thickness $\Omega \geq 1.5$, and H_1 is the average height of the trees in the central level of thickness.

Obviously, H_1 is the height of the average stem of the forest stand, the diameter of which is equal to d_{mean}.

The index of steepness is an analogue of increment percent of growth curves. The latter is widely used to study growth - of pure and mixed plantations, of conifers and broad-leaved trees, of tree species, etc. (Trouillier et al., 2020; Tomlinson et al. 2014, Seo et al, 2017).

Results

1. Relationships between the measures of skew

Figure 1 shows the relationship between the coefficient of asymmetry C (on the abscissa), and the parameters I, A and ZNI. There is a close relationship between the asymmetry coefficient C and the parameters I and A (R^2=0.93, and 0.94). No high correlation is observed between C and ZNI (the small crosses with the dotted linear trend line in the graph).

Note, that the scatter of I is entirely due to the above mentioned simplification. As A is function of I, its scatter simply mirrors that of I.

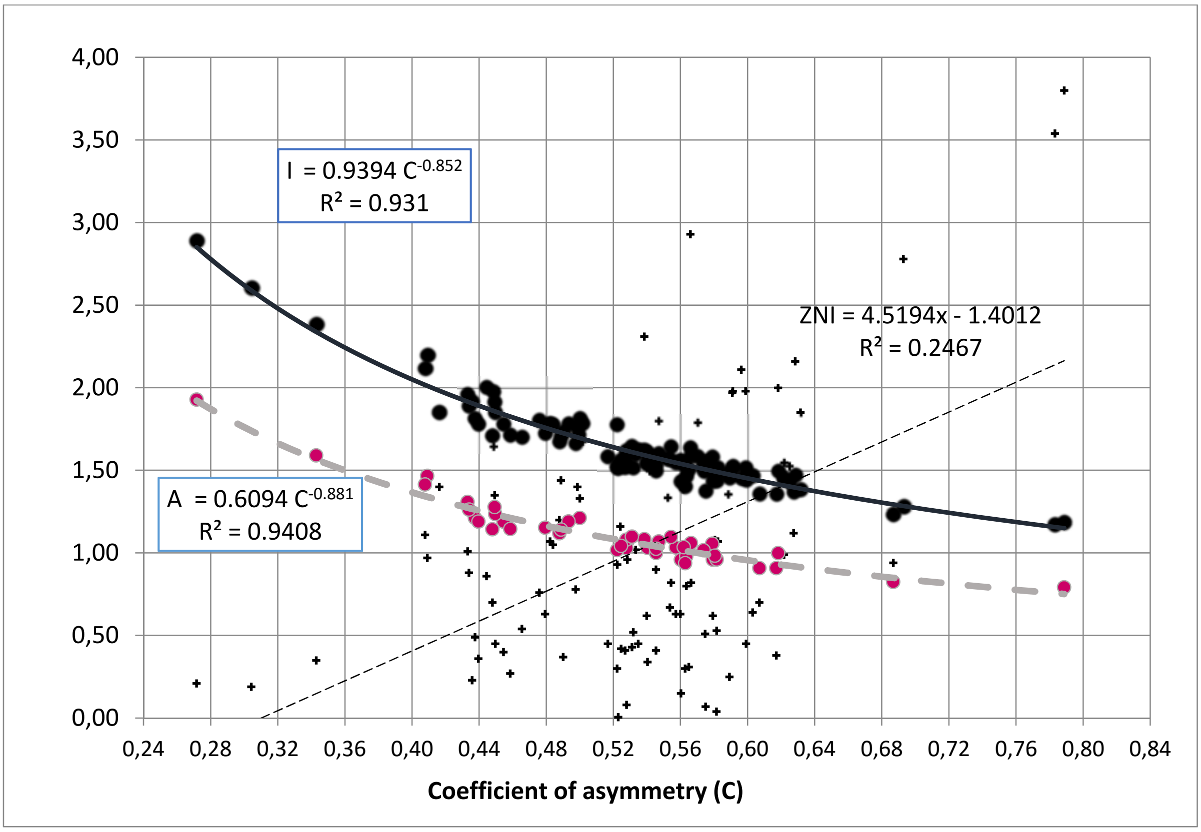

Figure 1. Relationships between the measures of asymmetry

Рисунок 1. Взаимосвязи между показателями асимметрии

Source: author's composition

Источник: собственная композиция автора

The figure shows that criteria C, I and A give the same answer to the question whether the asymmetry is left or right: as defined, the asymmetry is left if $C > 0.57$, $I < 1.5$ resp. A<1, and right otherwise. It can be seen that the trend lines of A and I reach their critical levels approximately at the point C=0.57. Contradictory decisions can only occur near this critical point due to the scatter of I and A points around their trendlines. Such cases will be few, because the scatter is weak. The ZNI trendline also reaches its critical level ZNI=1 about the point C=0,57. However, the scatter of the ZNI points is so great that this trend line is rather random, due to a few outliers. From the size and the shape of the cloud of ZNI points, it is seen that the criterion gives a different left/right and weak/strong judgments in half of the cases as compared with I and A.

2. *Values of the parameters*

Table 1 presents the distribution of the 101 sample plots according to types of asymmetry, the average values of the indicators C, I, A, ZNI and IS in the sample plots and values calculated from the Tyurin distribution in the rightmost column. The latter are given as a plausibility test.

Figure 2 compares the average diametrical distribution, obtained from the SPs, with the Tyurin distribution.

Table 1

Average values of the indicators in the SPs controlled with the Tyurin distribution

Таблица 1

Средние значения показателей на пробных площадях (контроль – распределение Тюрина)

Indicators	Sample plots data				Tyurin distribution
	Left skew	Right skew	No skew	**Total**	
Number of SPs	12	66	23	**101**	
Coefficient of Asymmetry (C)	0.66	0.5	0.58	**0.54**	**0.57**
Index of Asymmetry (I)	1.25	1.73	1.48	**1.63**	1.48
Asymmetry rate (A)	0.83	1.15	1.00	**1.09**	1.00
Zero Natural Indicator (ZNI)	0.83	1.89	1.15	**1.03**	1
Index of Steepness (IS)	1.09	1.05	1.08	**1.07**	1.10

Source: own calculations

Источник: собственные вычисления автора

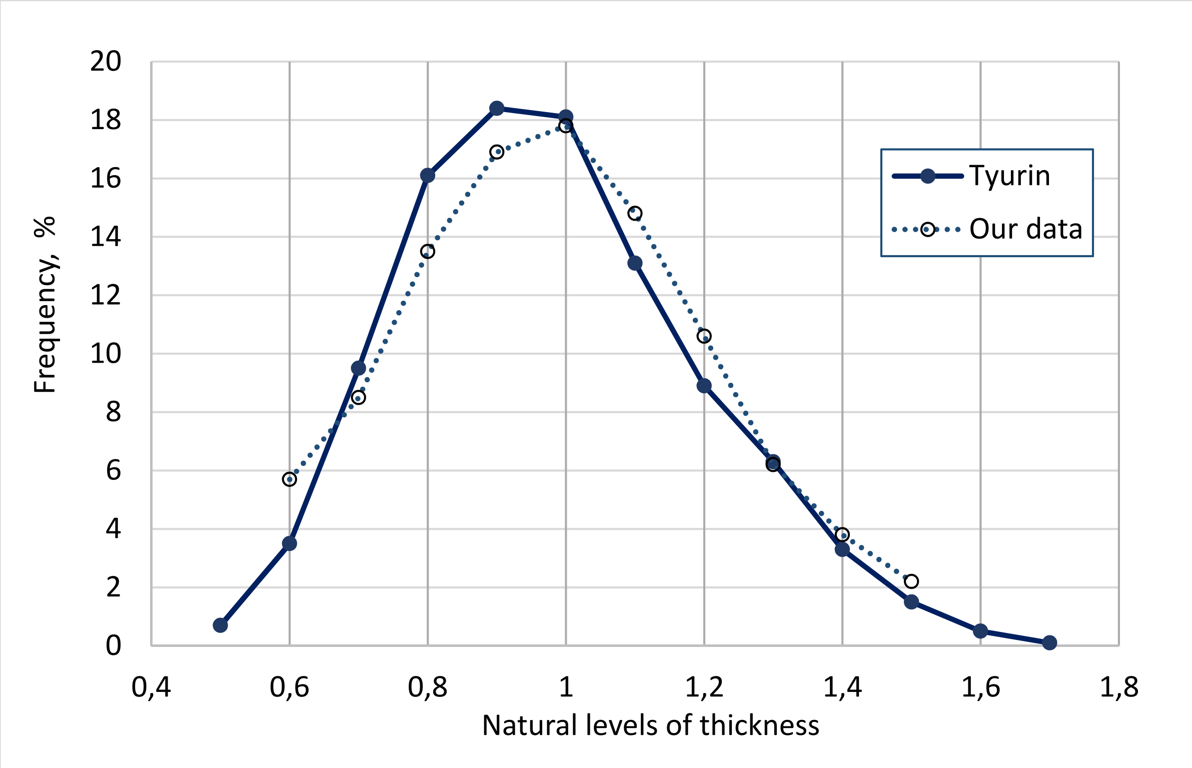

Figure 2. Average diameter distribution for our sample compared to the Tyurin distribution

Рисунок 2. Распределение среднего диаметра для исследуемой выборки по сравнению с распределением Тюрина

Source: author’s composition

Источник: собственная композиция автора

One can see in Table 1 that sample plots of a right-hand-side asymmetry (66) dominate, next followed by plots of the normal type (23) and plots of a left-hand-side asymmetric distribution (12). The average coefficients of asymmetry, the indices, the **asymmetry rate**, and the weighted average value of the zero natural indicators for all the sample plots are: 0.54, 1.63, 1.09 и 1.03, respectively. A lower index of steepness (I=1.05) – structure in height - corresponds to the right asymmetry (IS=1.15) – structure in thickness. Our average curve of distribution is slight right asymmetric (C=0.54, I=1.57), Tyurin’s Uniform Curve - Normal (Cas=0.57, I_{as}=1.48), which can be seen in fig. 1. The average index of steepness for the structure in height is 1.07, and this same index as calculated from The Uniform Average Curve of Height Structure (H) of Tyurin is 1.10, i.e. a less steep curve of heights corresponds to a thickness-structure asymmetry more deflected to the right (in our investigation).

3. Average Absolute and Relative Curves (Curves of the Normal Numbers) for the Structures in Thickness and Height

Transforming the distribution curves into a relative (q_x) type (Equation 7) makes it possible to decrease the interval within which the numerical values vary and to outline the asymmetry more clearly, and such relative curves (or the curves of the normal numbers) have been presented on Figure 3. It reveals that the **normal** curves stand at a distance from the abscissa close to the value of $q_{N1.0\_}$=1.0, and the curves of **the left-hand-side and right-hand-side asymmetries** jump up rather above 1 and place themselves on the left or on the right of the level of thickness 1.0.

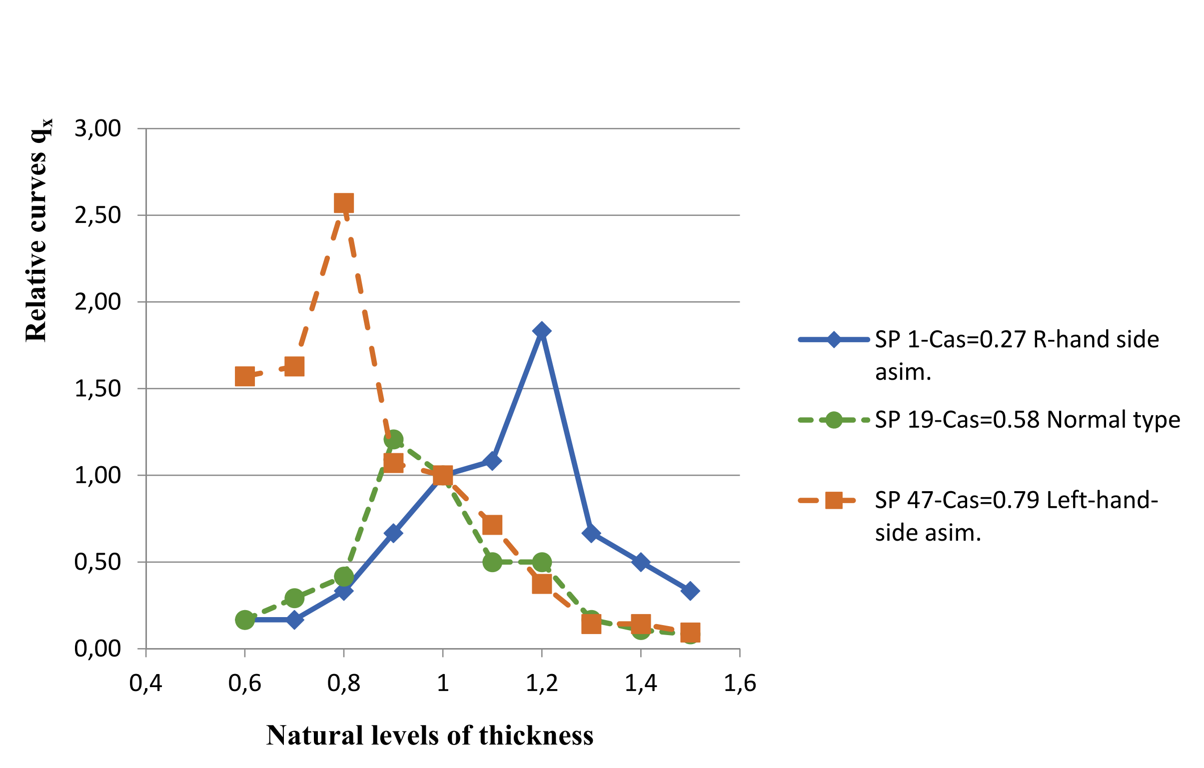

Figure 3. The three types of asymmetry presented by means of the relative curves q_x

Рисунок 3. Три типа асимметрии представлены с помощью относительных кривых q_x

The average and average relative curves of the structures in thickness ***d*** and height ***H***, as being of a common, characterising importance for the different aggregates of forest stands, are important parametric curves and are presented in Table 2, as according to our data so according to the generalised (Uniform) curves of Tyurin. One can also see the values of the indices obtained at the end of the thickness interval (1.4; 1.5).

Table 2

Average Parametric Curves of the Structures in Thickness and Height of Scots Pine (*Pinus sylvestris* L.) Stands

Таблица 2

Средние параметрические кривые структур по толщине и высоте древостоев сосны обыкновенной (*Pinus sylvestris* L.)

Average Curves	Natural levels of thickness									
	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Our data [R. Petrin]										
Numbers of trees N_x%	5.7	8.5	13.5	16.9	17.9	14.8	10.6	6.2	3.8	2.1
q_x (normal numbers)	0.32	0.47	0.76	0.94	1.00	0.83	0.59	0.34	0.21	0.12
$N_{x\_sum\_}$- Summed points' curve (SPC) %	5.7	14.2	27.7	44.6	62.5	77.3	87.9	94.1	97.9	100
q_{x_sum} (normal numbers of SPC)	0.09	0.23	0.44	0.71	1.00	1.24	1.41	1.51	***1.57***	***1.60***

q_{x_H} (normal numbers of height structure)	0.81	0.87	0.92	0.97	1.00	1.02	1.04	1.05	***1.07***	***1.12***
A.V. Tyurin's data										
q_x (normal numbers of *d curve*)	0.19	0.52	0.89	1.02	1.00	0.72	0.49	0.35	0.18	0.08
q_{x_sum} (normal numbers of SPC)	0.06	0.21	0.45	0.73	1.00	1.20	1.33	1.43	***1.48***	***1.50***
q_{x_H} (normal numbers of height structure)	0.85	0.89	0.93	0.97	1.00	1.03	1.06	1.08	***1.10***	***1.12***

Source: own calculations

Источник: собственные вычисления авторов

4. Asymmetry type of diametrical distribution and height curve steepness

Figure 4 shows the index of steepness set in relation to the asymmetry index of the same sample plot. The scatter is not large, but the dependence appears weak. The trend line shows that the steepness of the height curves decrease with the increase of the asymmetry index, i.e with the in the increase right asymmetry. An opposite trend has been found in oak stands at an average age of 95: the steepness of height curves increase with the increase in the right asymmetry (Petrin, R. 2021). A possible reason for this might be the age: the investigated Scots pine forests are younger, their average age being 72. With thickness levels lower than the average diameter $\boldsymbol{d_{av}}$, the younger stands grow in height faster than the older ones, i.e. the right asymmetry in the distribution of tree numbers according to thickness does not lead to increase in the steepness of height curves.

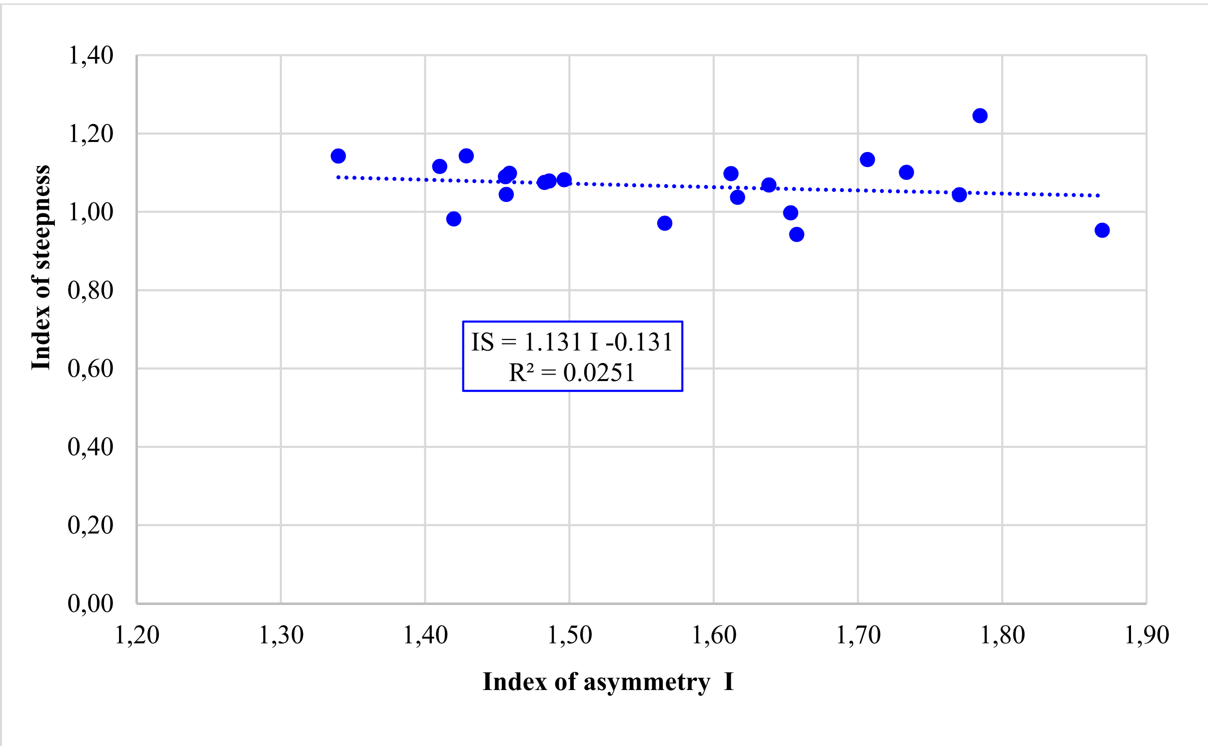

Figure 4. Dependence of the index of steepness (IS) on the index of asymmetry (I)

Рисунок 4. Зависимость показателя крутизны (IS) от показателя асимметрии (I)

Discussion

Published research. Worldwide, the shape of the diametrical distribution curves has been the subject of numerous studies. In Bulgaria, many studies focuse on the relationships of asymmetric distributions with stand parameters (Nedyalkov, 1955) or cuts (Krastanov, 1968). Many case studies established empirically the skew for different tree species and forest types e.g. the author in a series of studies. The variety of skew is often limited to three asymmetry types: symmetric (or normal), right asymmetric and left asymmetric (E. P. Dimitrov, 1978). So did E. T. Dimitrov (2003), while fitting volumetric models for Scots pine, Norway spruce and silver fir. In the investigations cited, the statistical asymmetry coefficient $\gamma = \mu_3/\sigma^3$, or the ZNI were used. Because of different methodology, the results obtained are not directly comparable, incl. with our investigation, mentioned in the introduction. Recent research on diameter distribution has focused on fitting the Weibull equation from stand-level variables (Stankova and Rodriguez-Aranda, 2010; Stankova and Zlatanov, 2010).

Definitions. Formula (2) is derived as follows: In statistics, the approximate formula for estimating asymmetry is known $\beta_3 = \sqrt{2\pi}(0.5 - C)$, where β_3 is the Charlier coefficient and $C = P\left[d \le \bar{d}\right]$ is the frequency of diameters thinner than the *arithmetic* mean diameter $\bar{d}$ (Prodan, 1961, 1965). For β_3 it is valid $\beta_3 = -\gamma/6$ (Prodan, 1961). It follows a linear relationship with constant coefficients between γ and C. This is sufficient to know that

$$C = P\left[d \le \bar{d}\right] \quad (10)$$

is a measure of skewness. We prefer C to the quantity β_3, first, for simplicity, and second, because it is a strictly positive quantity, such as is preferred in forestry practice.

Finally, formula (10) differs from formula (2), which we use here, in that in even-age forest stands with a normal distribution, the coefficient of asymmetry C is about 60% (Turin, 1930), not 50%. Therefore, when studying the structure according to thickness formula (10) gives an insufficiently accurate result.

Results. The strong ZNI scattering in Fig. 1 is an expected result. The ZNI postulates a linear relationship between the standard and deviating curves, a condition that is often not met.

The weak slope of the trend line in Fig. 3 and the existence of case studies with a reverse slope speak rather that there is no correlation between I and IS. If so, these factors should be examined independently.

Conclusion

The indices and indicators of asymmetry characterise the asymmetry in thickness structure much more precisely than the zero natural indicators and can be used for making generalised, comparative investigations as, for example, for finding out the relationship between the structures in thickness and height or the other kinds of structure of forest dendrocoenoses.

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