



Summary Comparative Studies on the Relationship Between Thickness and Height Structures of Stands of Different Tree Species

Rumen Dimitrov Petrin ✉ lesni4eja2014@gmail.com

Forest Research Institute of the Bulgarian Academy of Sciences, 132 Kliment Ohridski Blvd., Sofia, 1756, Bulgaria

Abstract

This study summarizes twelve years of research on the structure of forest stands of different tree species, focusing on the relationship between diameter distributions (thickness structure) and height–diameter curves (height structure). Based on numerous sample plots, the analysis compares distribution curves of the number of trees by diameter classes relative to the mean diameter (d_{av}), and corresponding average height curves. Two newly introduced indicators – the asymmetry index (I_{as}) for thickness structure and the steepness index (I_{st}) for height structure – provide simple and practical tools for analyzing the dynamics of stand structure. Results demonstrate that all studied species (spruce, fir, Scots pine, seed-origin beech, seed-origin oak, coppice oak, and Scots pine plantations) show right-skewed asymmetry in diameter distribution, which corresponds to steeper height curves. Comparisons with Tyurin’s Unified Curve (1938) confirm the validity of these findings. The relationship between asymmetry and steepness makes it possible to evaluate stand productivity and wood volume potential more effectively.

Keywords: *stand structure, diameter distribution, asymmetry index, steepness index, tree species, stand productivity, wood volume*

For citation: Petrin R. D. Summary Comparative Studies on the Relationship Between Thickness and Height Structures of Stands of Different Tree Species (2026). Forestry Engineering journal, Vol. 16, No. 1 (61), pp. 7–19. DOI: <https://doi.org/10.34220/issn.2222-7962/2026.1/1>.

Received 23.12.2025. *Revised* 29.01.2026. *Accepted* 15.03.2026. *Published online* 27.03.2026.

Introduction

The regularities in the structure of forest stands form the basis for creating adequate models for assessing the amount of timber in the forest and its assortment structure. The present author, over a period of 12 years, using a sufficient number of sample plots (SP) for stands of various tree species, has studied the distribution curves of the number of trees (N) by diameter classes to establish the symmetry or asymmetry in this distribution relative to the average diameter d_{av} . On the basis

of the same sample plots, height curves were also studied, representing average heights (H_{av}) by diameter classes for a given stand. The results have been published in 13 publications [10–22].

In this work, an attempt is made to summarize the results of these studies by comparing two types of stand structures – by thickness (N_d) and by height (H) – for the following tree species (TS): spruce, fir, Scots pine, seed-origin beech, seed-origin oak, coppice oak, and Scots pine plantations.

The distribution curves are studied in relation to the average diameter (\underline{d}_{av}), which according to forest inventory science (Weise's rule) for even-aged stands divides the total number of trees into two unequal parts: about 60% thinner and 40% thicker. When we have 57% thinner trees, the distribution is considered symmetrical (based on A. Tyurin's Unified Curve, 1938). Right asymmetry occurs when the number of trees thicker than \underline{d}_{av} predominates, which is advantageous from the viewpoint of wood volume and assortment structure. Steepness, on the other hand, characterizes the height curves (H) and shows the magnitude of height H increase between diameter classes; under otherwise equal conditions, stands with steeper curves are more productive.

Da Silva et al. (2024), using NLME (Nonlinear Mixed-Effects) models, established the parameters of the height–diameter (H–D) relationships at different stand densities. The steepness index (I_{st}) (Petřin, 2021, 2022, 2023a, 2024, 2025) can be used as a simpler and more accessible alternative to the steepness parameters bbb , reflecting the influence of competition (stand density) on the steepness of the height curves.

Ciceu et al. (2020, 2023), studying H–D models for the structure of broadleaved tree species, found that environmental conditions affect the shape and steepness of height curves. I_{st} allows direct comparison of this dynamic without complex parameterization.

Bronisz & Mehtätalo (2020), with hierarchical NLME models, show that competition and stand density influence the shape (steepness) of H–D curves. I_{st} is functionally similar to their steepness indicators and can also serve as a simpler and more directly computable measure in similar studies.

In mixed stands, Siipilehto et al. (2023) investigated the shape of H–D curves depending on species interactions. I_{st} can summarize these differences, providing the possibility for a practical forestry interpretation.

MacPhee et al. (2018) found that H–D model parameters reflect the rate of height development depending on stand density. I_{st} represents a direct and simpler alternative for comparing curves at different degrees of competition.

The Weibull function, used in studies by Bailey & Dell (1996), describes diameter distributions through the shape parameter kkk , which indicates the degree of asymmetry. The

asymmetry index (I_{as}) of Petřin (2023, 2024), which is directly calculated from the cumulative curve, is analogous to kkk .

Ciceu, Pitar, and Badea (2021) analyzed diameter distributions in mixed, uneven-aged stands in the Southern Carpathians (Romania). The main objective was to develop statistical models describing the structure of diameter classes and their dynamics under varying age and spatial heterogeneity. The authors used flexible probability functions, including Weibull, to describe the actual distribution of tree numbers by diameter and to assess their accuracy. It was shown that in uneven-aged stands, distributions are often asymmetric, skewed toward smaller diameters. This tendency can also be interpreted using Petřin's asymmetry index, which numerically captures deviations of the distribution from symmetry.

In their study, Lee, D.; Siipilehto, J.; Miina, J.; Niemistö, P.; Haapanen, M.; Hynynen, J.; Huuskonen, S. (2024) used a site index and models of stand characteristics for silver birch plantations in southern and central Finland. By analyzing heights and diameters in plantation conditions, they showed that tree size distributions are often approximately normal or slightly skewed. Here, asymmetry is linked to productivity assessment. In fast-growing species, the distribution is more balanced (Weibull $k \approx 3.5-4.0$ \approx 3.5–4.0, i.e., slight negative skew), meaning that Petřin's asymmetry index (2024, 2025) would have moderate values. On poorer soils and in more uneven stands, a clearer positive skew appears (trees predominantly on the left of the average, leading to lower values of Petřin's asymmetry indices).

Lebedev, A. V.; Kuzmichev, V. V. (2020) tested the applicability of simple 2- and 3-parameter models for the height–diameter relationship of birch (*Betula pendula* and *Betula pubescens*) in the European part of Russia. The main models used were Monamur, Korf, Näslund, etc. The authors recommend using such models for forest management purposes and for stock predictions.

Lebedev, A. V. (2020), based on a large number of sample plots of varying age, site quality, and density, developed new generalized $h=f(d)$ $h=f(d)$ models for birch stands in European Russia. He introduced generalized nonlinear functions valid for all stands.

Objects and Methods

The author's publications on the stands of the above-mentioned tree species are listed in the References.

The formulas used in the study are as follows:

Relative curves q_x , two types:

1. Distribution curves of the number of trees:

$$q_{Nx} = N_k / N_{d_{av}} \quad (1),$$

where N_k is the number of trees in the respective diameter class, and $N_{d_{av}}$ – the number of trees corresponding to the mean diameter.

2. Height curves:

$$q_{Hx} = H_k / H_{av} \quad (2),$$

where H_k is the height in the respective diameter class, and

H_{av} – the average height of the stand.

Cumulative distribution curve
Regardless of the type of distribution – relative or absolute – the cumulative distribution curve is calculated, which represents a constantly increasing

The asymmetry coefficient represents THE RATIO of the number of trees up to the central diameter class (d_{av}) plus half of the number corresponding to it, to the total number (in relative form – 100% or 1.0).

Asymmetry index of the distribution

$$I_{as} = N_{sum_{\Omega}} / N_{sum_m} \quad (5),$$

where:

$N_{sum_{\Omega}}$ – the total number of trees (which in the cumulative curve corresponds to the final right-hand diameter class), and
 N_{sum_m} – the cumulative number of trees up to and including the mean diameter d_{av} .

Steepness index of the height structure curves

$$I_{st} = H_{\Omega} / H_m \quad (6), \text{ where:}$$

semi-parabolic curve, similar to the usual form of the height curve.

The formula is:

$$N_{sum_x} = N_k + \sum N_{1+k} \quad (3), \text{ where:}$$

N_k is the number of trees in a given diameter class k , and $\sum N_{1+k}$ – the sum of the numbers of trees in the preceding diameter classes.

The cumulative curve is obtained so that for each diameter class the sum corresponds to the number of trees in all classes up to and including the current one.

Asymmetry coefficient

$$Cas = (\sum N_{1+k} + N_{d_{av}} / 2) / \sum N_x \quad (4),$$

where:

$\sum N_{x_{1+k}}$ – the sum of the numbers of trees (N_x) up to the diameter class corresponding to d_{av} ,

$N_{d_{av}}$ – the number of trees in the central diameter class,

$\sum N_x$ – the total number of trees in the stand.

H_{Ω} – the height at the end of the studied diameter interval, and

H_m – the height at d_{av} , i.e. the average height of the stand.

Results

In Table 1 the curves and the two asymmetry indicators – the asymmetry coefficient and the asymmetry index – are presented for each tree species. For spruce, for example, the distribution curve is shown first in relative (corresponding to percentage) form, followed by q_x of the same curve (from formula 1), then the cumulative curve is calculated (formula 3), and finally q_x of the cumulative curve or q_x of the asymmetry (again according to formula 1). The average relative distribution curve for all tree species has also been calculated, and similar calculations have been carried out for Tyurin's unified distribution curve.

Table 1

Calculation of asymmetry indicators coefficients and indices of asymmetry

Tree Species	Indicator s	Natural Degrees of Thickness											Asymmetry Coefficient (Cas)
		0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	
Spruce	Nrel	0.07	0.09	0.11	0.14	0.15	0.15	0.11	0.08	0.05	0.03	0.02	0.49

Естественные науки и лес

	qx_N	0.4 7	0.6 0	0.7 3	0.9 3	1.0 0	1.0 0	0.7 3	0.5 3	0.3 3	0.2 0	0.1 3	
	Nsum	0.4 7	1.0 7	1.8 0	2.7 3	3.7 3	4.7 3	5.4 7	6.0 0	6.3 3	6.5 3	6.6 7	
	qx_asi m	0.1 3	0.2 9	0.4 8	0.7 3	1.0 0	1.2 7	1.4 6	1.6 1	1.7 0	1.7 5	1.7 9	
Fir	Nrel	0.0 8	0.1 0	0.1 4	0.1 5	0.1 4	0.1 3	0.0 9	0.0 8	0.0 5	0.0 3	0.0 1	0.54
	qx_N	0.5 7	0.7 1	1.0 0	1.0 7	1.0 0	0.9 3	0.6 4	0.5 7	0.3 6	0.2 1	0.0 7	
	Nsum	0.5 7	1.2 9	2.2 9	3.3 6	4.3 6	5.2 9	5.9 3	6.5 0	6.8 6	7.0 7	7.1 4	
	qx_asi m	0.1 3	0.3 0	0.5 2	0.7 7	1.0 0	1.2 1	1.3 6	1.4 9	1.5 7	1.6 2	1.6 4	
Scots Pine	Nrel	0.0 6	0.0 8	0.1 4	0.1 7	0.1 8	0.1 5	0.1 1	0.0 6	0.0 4	0.0 2	0.0 1	0.53
	qx_N	0.3 3	0.4 4	0.7 8	0.9 4	1.0 0	0.8 3	0.6 1	0.3 3	0.2 2	0.1 1	0.0 6	
	Nsum	0.3 3	0.7 8	1.5 6	2.5 0	3.5 0	4.3 3	4.9 4	5.2 8	5.5 0	5.6 1	5.6 7	
	qx_asi m	0.1 0	0.2 2	0.4 4	0.7 1	1.0 0	1.2 4	1.4 1	1.5 1	1.5 7	1.6 0	1.6 2	
Seed Beech	Nrel	0.0 8	0.1 1	0.1 4	0.1 4	0.1 4	0.1 2	0.1 0	0.0 7	0.0 5	0.0 4	0.0 2	0.54
	qx_N	0.5 6	0.7 5	1.0 0	1.0 0	1.0 0	0.8 8	0.6 9	0.5 0	0.3 8	0.2 5	0.1 3	
	Nsum	0.5 6	1.3 1	2.3 1	3.3 1	4.3 1	5.1 9	5.8 8	6.3 8	6.7 5	7.0 0	7.1 3	
	qx_asi m	0.1 3	0.3 0	0.5 4	0.7 7	1.0 0	1.2 0	1.3 6	1.4 8	1.5 7	1.6 2	1.6 5	
Seed Oak	Nrel	0.0 6	0.1 0	0.1 3	0.1 7	0.1 7	0.1 5	0.1 0	0.0 6	0.0 3	0.0 2	0.0 1	0.54
	qx_N	0.3 6	0.5 8	0.7 5	1.0 4	1.0 0	0.9 0	0.6 0	0.3 5	0.1 9	0.1 3	0.0 6	
	Nsum	0.3 6	0.9 4	1.6 9	2.7 2	3.7 2	4.6 3	5.2 3	5.5 7	5.7 6	5.8 9	5.9 5	
	qx_asi m	0.1 0	0.2 5	0.4 5	0.7 3	1.0 0	1.2 4	1.4 0	1.5 0	1.5 5	1.5 8	1.6 0	
Coppice Oak	Nrel	0.0 6	0.0 9	0.1 2	0.1 7	0.1 7	0.1 6	0.1 0	0.0 6	0.0 3	0.0 2	0.0 1	0.53
	qx_N	0.3 5	0.5 4	0.6 9	1.0 0	1.0 0	0.9 2	0.6 0	0.3 5	0.1 9	0.1 3	0.0 6	
	Nsum	0.3 5	0.8 8	1.5 8	2.5 8	3.5 8	4.5 0	5.1 0	5.4 4	5.6 3	5.7 7	5.8 3	
	qx_asi m	0.1 0	0.2 5	0.4 4	0.7 2	1.0 0	1.2 6	1.4 2	1.5 2	1.5 8	1.6 1	1.6 3	
Scots Pine Plantation s	Nrel	0.0 7	0.1 0	0.1 3	0.1 5	0.1 5	0.1 5	0.1 0	0.0 6	0.0 4	0.0 3	0.0 2	0.52
	qx_N	0.4 4	0.6 3	0.8 8	1.0 0	1.0 0	1.0 0	0.6 9	0.3 8	0.2 5	0.1 9	0.1 3	
	Nsum	0.4 4	1.0 6	1.9 4	2.9 4	3.9 4	4.9 4	5.6 3	6.0 0	6.2 5	6.4 4	6.5 6	
	qx_asi m	0.1 1	0.2 7	0.4 9	0.7 5	1.0 0	1.2 5	1.4 3	1.5 2	1.5 9	1.6 3	1.6 7	
Average	qx_N	0.4 4	0.6 1	0.8 3	1.0 0	1.0 0	0.9 2	0.6 5	0.4 3	0.2 7	0.1 8	0.0 9	0.53

Average Tyurin Curve	NotH.	0.0 4	0.1 0	0.1 6	0.1 8	0.1 8	0.1 3	0.0 9	0.0 6	0.0 3	0.0 2	0.0 1	0.57
	qx_N	0.1 9	0.5 2	0.8 9	1.0 2	1.0 0	0.7 2	0.4 9	0.3 5	0.1 8	0.0 8	0.0 3	
	Nsum	0.1 9	0.7 2	1.6 1	2.6 2	3.6 2	4.3 5	4.8 4	5.1 9	5.3 7	5.4 5	5.4 8	
	qx__asi m	0.0 5	0.2 0	0.4 4	0.7 2	1.0 0	1.2 0	1.3 4	1.4 3	1.4 8	1.5 0	1.5 1	

*- The relative expression of the number of trees N_{rel} corresponds to the percentage.

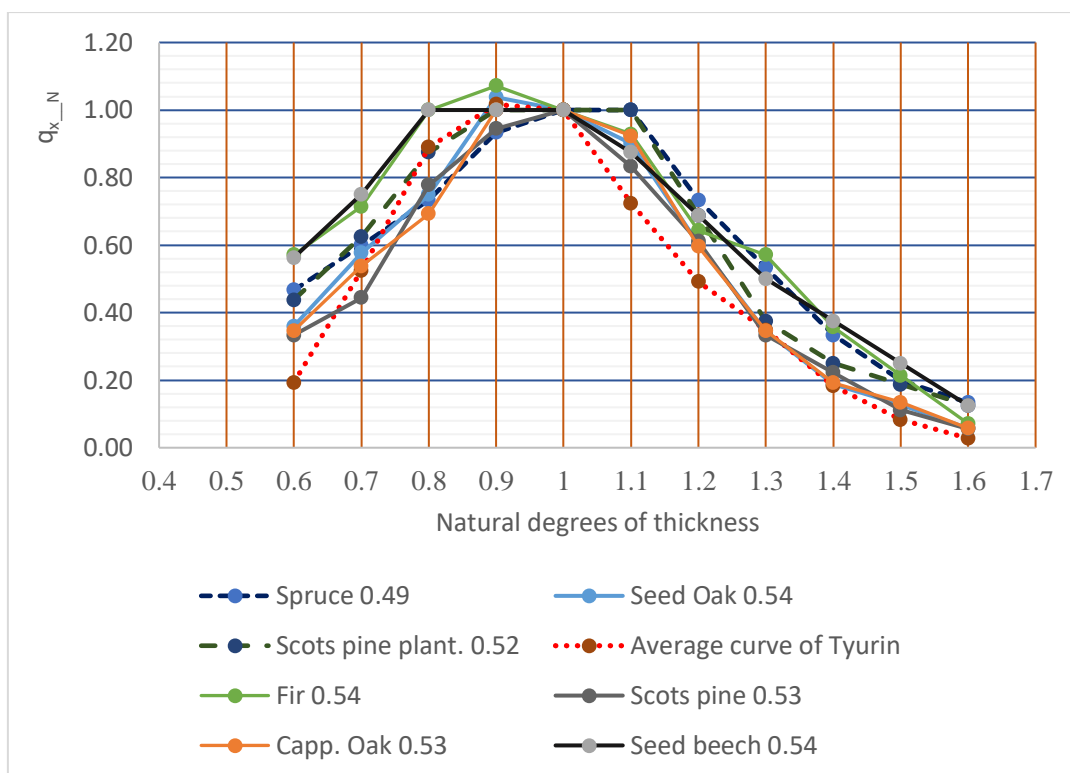


Figure 1. Relative distribution curves - q_{x_N} by tree species

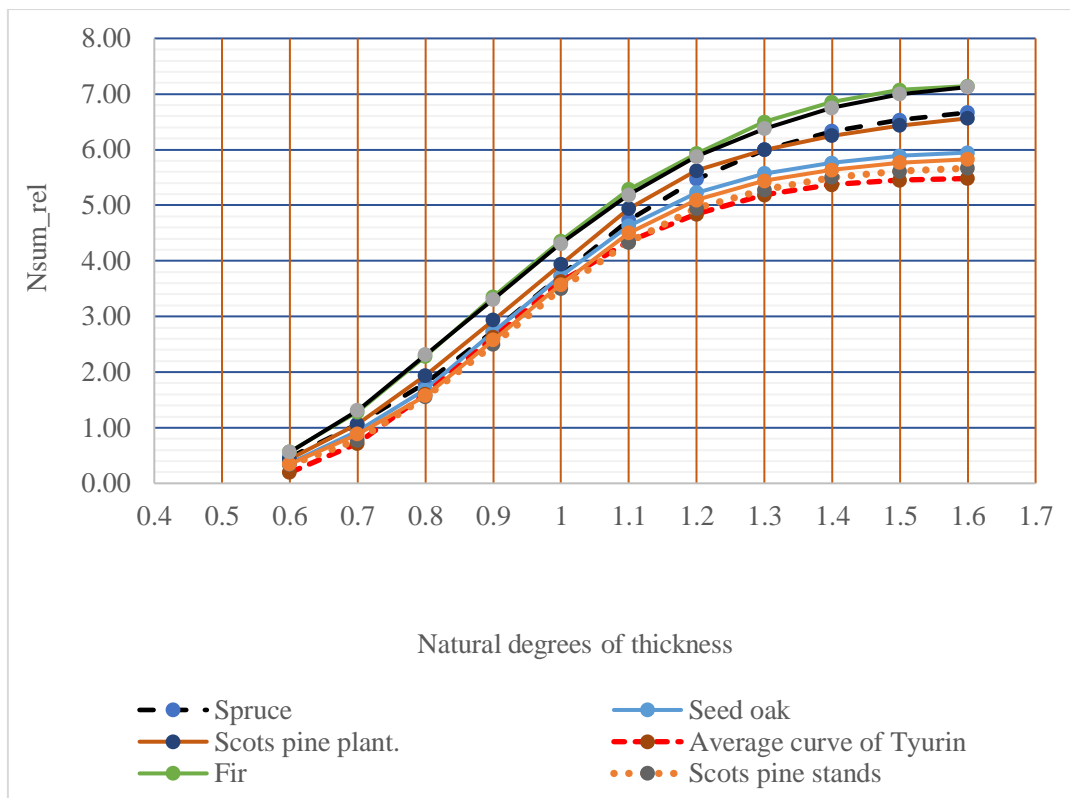


Figure 2. Summary (relative) distribution curves - N_{sum_rel} by tree species

As can be seen from Fig. 2, the cumulative distribution curves are of a form similar to the height curves – rising semi-parabolic curves. (That is why the formulas for I_{as} (f. 3) and for I_{st} (f. 6) are similar in nature.) From the cumulative curves themselves, their steepness cannot be estimated.

All distribution curves (q_x), which are relative with respect to the number of trees corresponding to the mean diameter (NDT = 1.0) , intersect at the abscissa and ordinate point 1.0, and the figure shows the differences in their asymmetry with respect to d_{av} . The corresponding asymmetry

coefficients indicate that the difference in asymmetry is not large for the tree species. That is, all tree species – spruce, fir, Scots pine, seed beech, seed oak, and coppice oak – have very close **ranks of the mean tree**, for which the asymmetry coefficient is the indicator; it varies from 0.49 for spruce to 0.54 for seed beech, oak, and fir. Thus, they all have a right-skewed distribution. The average asymmetry coefficient is $C_{as} = 0.53$, while that of **Tyurin’s unified curve equals 0.57**. Therefore, relative curves were calculated, called Normal Number Curves, or asymmetry curves, which are presented in Table 2.

Table 2

Curves of normal numbers of summary curves, or asymmetry curves - q_{x_asim}

Tree species	Natural degrees of thickness										
	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6
Spruce	0,13	0,29	0,48	0,73	1,00	1,27	1,46	1,61	1,70	1,75	1,79
Fir	0,13	0,30	0,52	0,77	1,00	1,21	1,36	1,49	1,57	1,62	1,64
Scots pine	0,10	0,22	0,44	0,71	1,00	1,24	1,41	1,51	1,57	1,60	1,62

Seed beech	0,13	0,30	0,54	0,77	1,00	1,20	1,36	1,48	1,57	1,62	1,65
Seed oak	0,10	0,25	0,45	0,73	1,00	1,24	1,40	1,50	1,55	1,58	1,60
Cappice oak	0,10	0,25	0,44	0,72	1,00	1,26	1,42	1,52	1,58	1,61	1,63
Scots pine pl.	0,11	0,27	0,49	0,75	1,00	1,25	1,43	1,52	1,59	1,63	1,67
Average	0,11	0,27	0,48	0,74	1,00	1,24	1,41	1,52	1,59	1,63	1,66
Average curve of Tyurin	0,05	0,20	0,44	0,72	1,00	1,20	1,34	1,43	1,48	1,50	1,51

From Table 2 and Fig. 3 it can be seen that the **asymmetry curves** q_{x_as} intersect at the same abscissa point, corresponding to NDT 1.0, i.e. d_{av} ,

while at the final abscissa point NDT=1.6 the asymmetry index is recorded for the individual tree species.

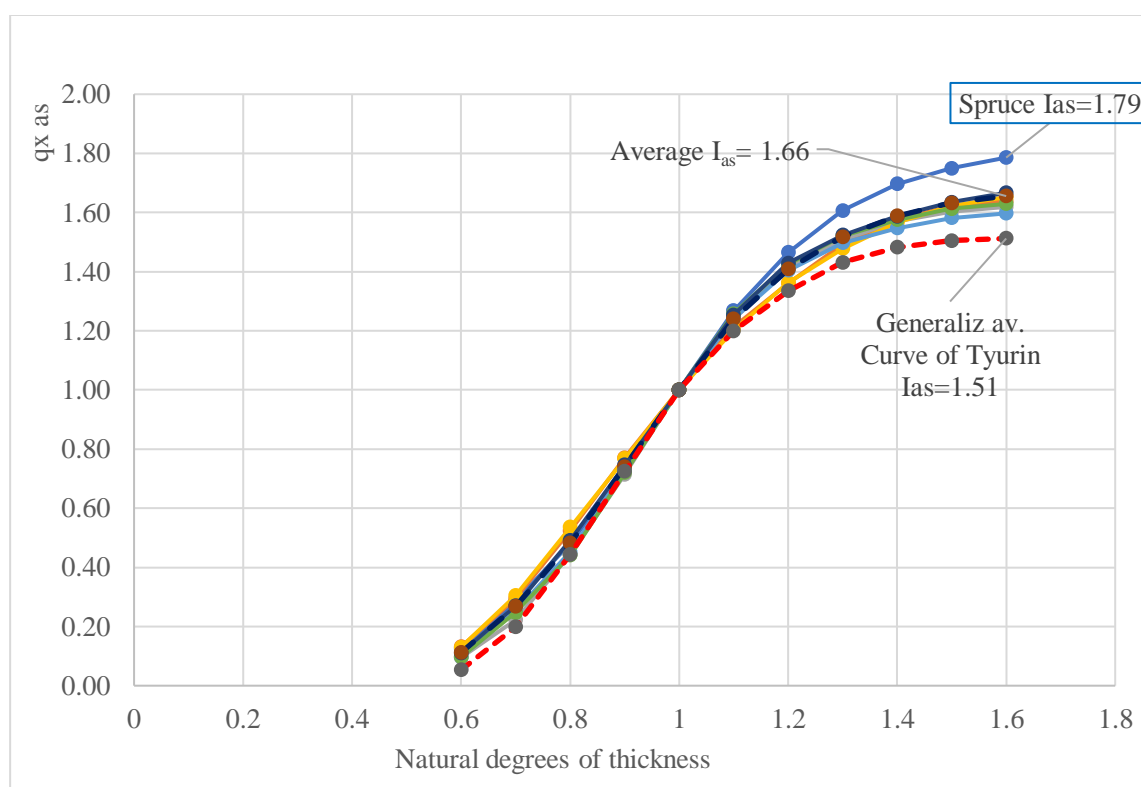


Figure 3. Curves of normal numbers q_{x_asim} of asymmetry

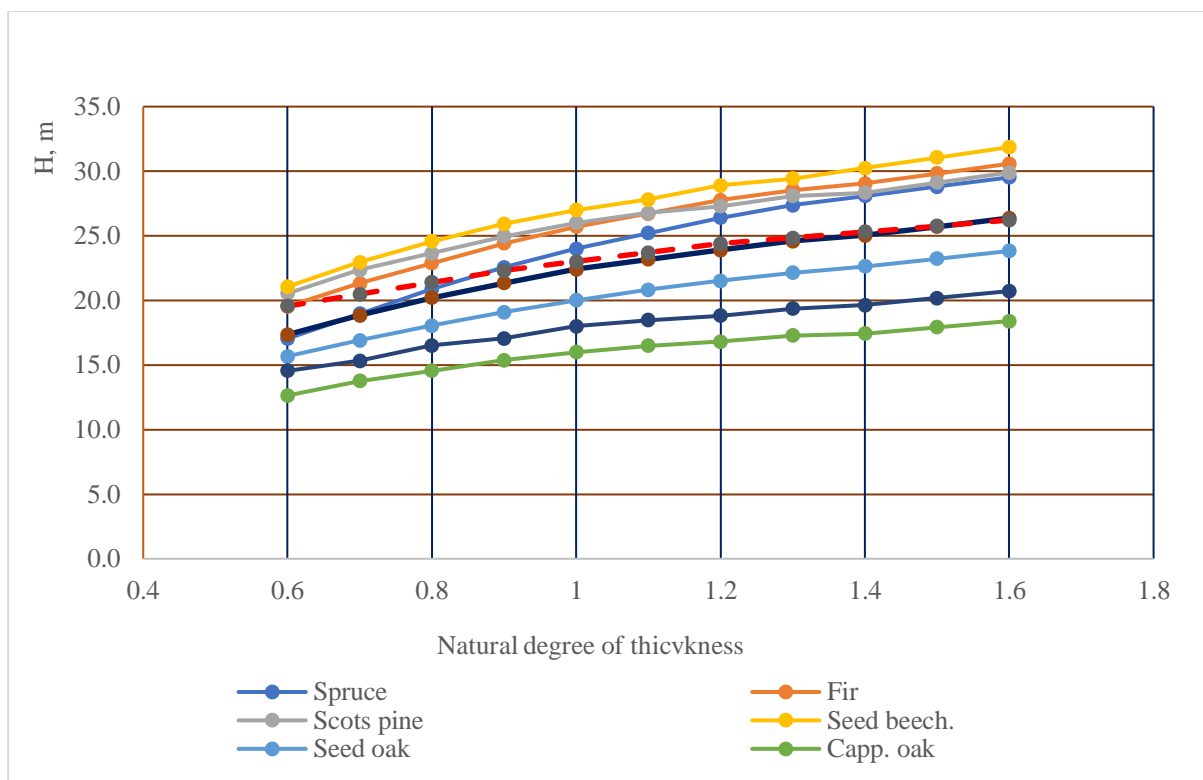


Figure 4. Average absolute height curves by tree species

In Fig. 3 the most characteristic values of the asymmetry indices are shown – the highest is I_{as} of spruce, with the most right-skewed asymmetry – 1.79, the average is 1.66, and the lowest is that of Tyurin’s unified curve – 1.51, which corresponds to the SYMMETRICAL type of distribution. The larger the asymmetry index, the more right-skewed it is.

In Fig. 4 the height curves by tree species in meters are shown, also called height structure curves. From the absolute height curves one cannot judge their STEEPNESS. Therefore, relative height curves were calculated in relation to the average height of the respective stands.

Table 3

Average relative height curves by tree species - $q_x H$

Tree species	Natural degrees of thickness										
	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6
Spruce	0,71	0,79	0,87	0,94	1,00	1,05	1,1	1,14	1,17	1,20	1,23
Fir	0,76	0,83	0,89	0,95	1,00	1,04	1,08	1,11	1,13	1,16	1,19
Scots pine	0,79	0,86	0,91	0,96	1,00	1,03	1,05	1,08	1,09	1,12	1,15
Seed beech	0,78	0,85	0,91	0,96	1,00	1,03	1,07	1,09	1,12	1,15	1,18
Seed oak	0,78	0,85	0,90	0,95	1,00	1,04	1,08	1,11	1,13	1,16	1,19
Cappice oak	0,79	0,86	0,91	0,96	1,00	1,03	1,05	1,08	1,09	1,12	1,15
Scots pine pl.	0,81	0,85	0,92	0,95	1,00	1,03	1,04	1,08	1,09	1,12	1,15

Average	0,77	0,84	0,90	0,95	1,00	1,04	1,07	1,10	1,12	1,15	1,18
Average curve of Tyurin	0,85	0,89	0,93	0,97	1,00	1,03	1,06	1,08	1,10	1,12	1,14

In Table 3 and in Fig. 5 the relative height curves q_{x_H} are presented numerically and also illustrated, which again intersect at NDT 1.0. In the last column of the table the steepness indices of the height curves can be seen. And in Fig. 5 it is shown that the steepest height curve is that of spruce, with the highest steepness index – $I_{st} = 1.23$, the average index is 1.18, and the lowest is for Tyurin’s unified height curve – 1.14, i.e. there is complete

correspondence between the asymmetry indices (Fig. 3) and the steepness indices.

Figure 6 shows three the mean relative curves - of distribution, asymmetry (from the cumulative distribution), and height steepness for all tree species combined, together with Tyurin’s corresponding curves. They intersect at the same abscissa-ordinate point (1.0), enabling direct comparison. Our results indicate higher asymmetry (greater right-hand skewness) and correspondingly higher steepness indices of the height curves (1.66; 1.18) than those of Tyurin (1.51; 1.14).

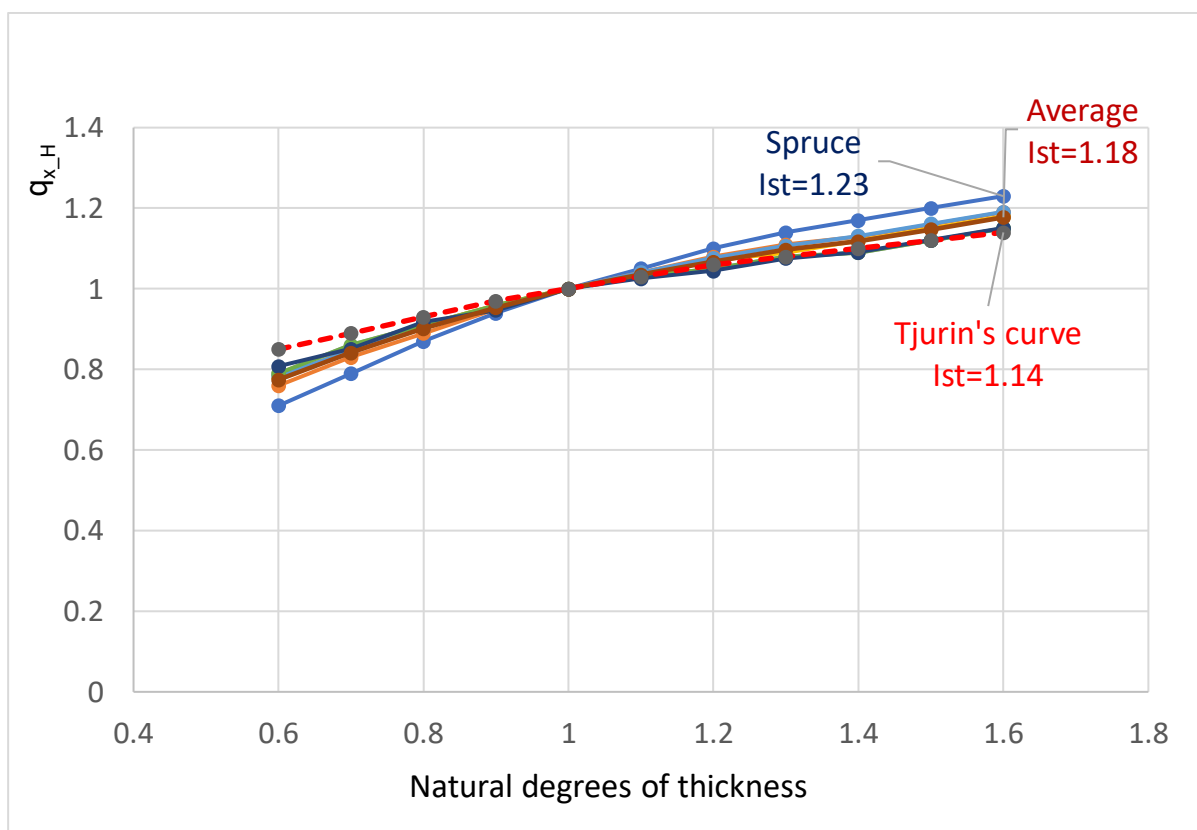


Figure 5. Average relative height curves q_{x_H} by tree species and I_{st}

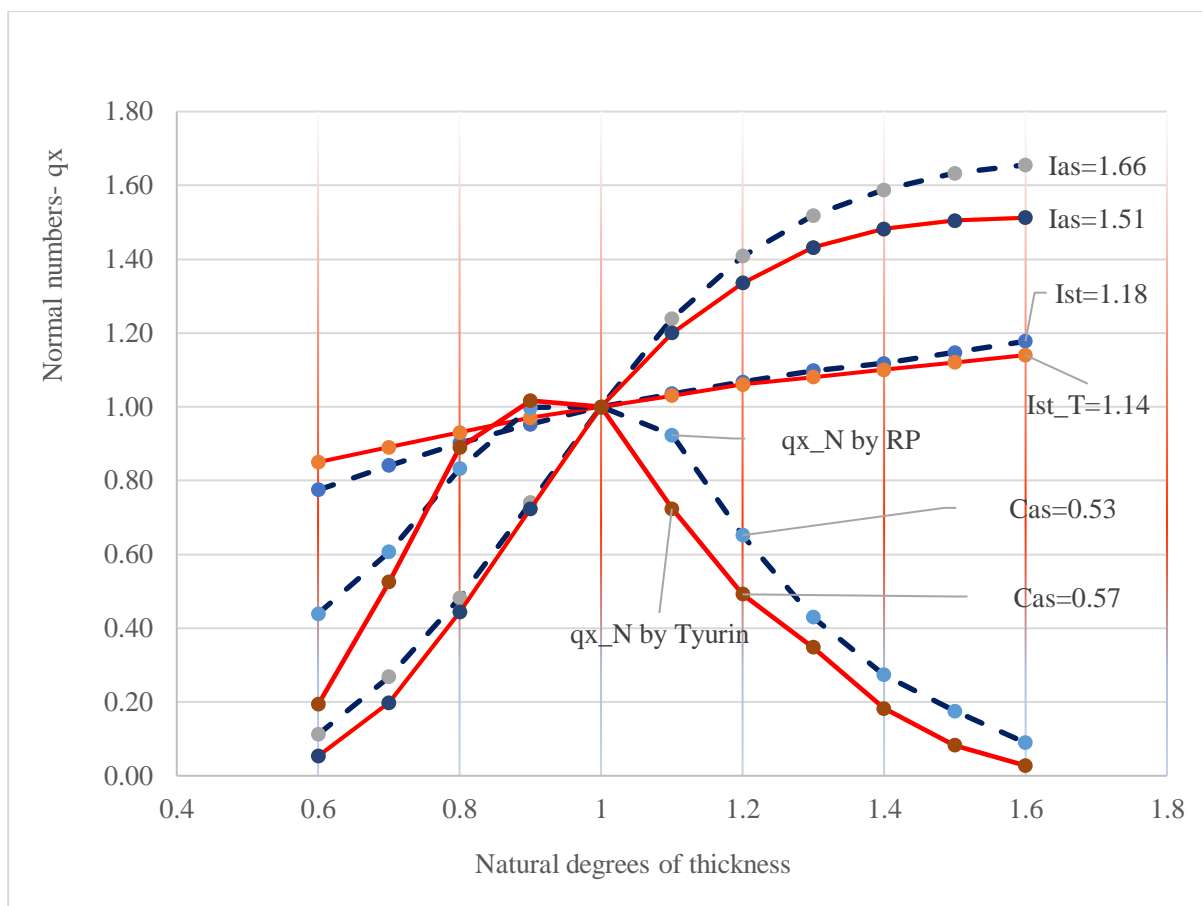


Figure 6. Average curves of distribution (N_d), asymmetry (q_{x_N}), and height (q_{x_H}) – ours and Tyurin's

Table 4

Summary data for C_{as} , I_{as} and I_{st_H}

Tree Species	Cas	Ias	Ist_H
Spruce	0.49	1.79	1.23
Scots Pine Plantations	0.52	1.67	1.15
Coppice Oak	0.53	1.63	1.15
Scots Pine	0.53	1.62	1.15
Seed Beech	0.54	1.65	1.18
Fir	0.54	1.64	1.19
Seed Oak	0.56	1.60	1.19
Average	0.53	1.64	1.17
Average curve of Tyurin	0.57	1.5	1.14

In Table 4 the values of the asymmetry coefficients, and correspondingly the asymmetry indices and the steepness indices, are presented in

ascending order. The same data are illustrated in Fig. 7.

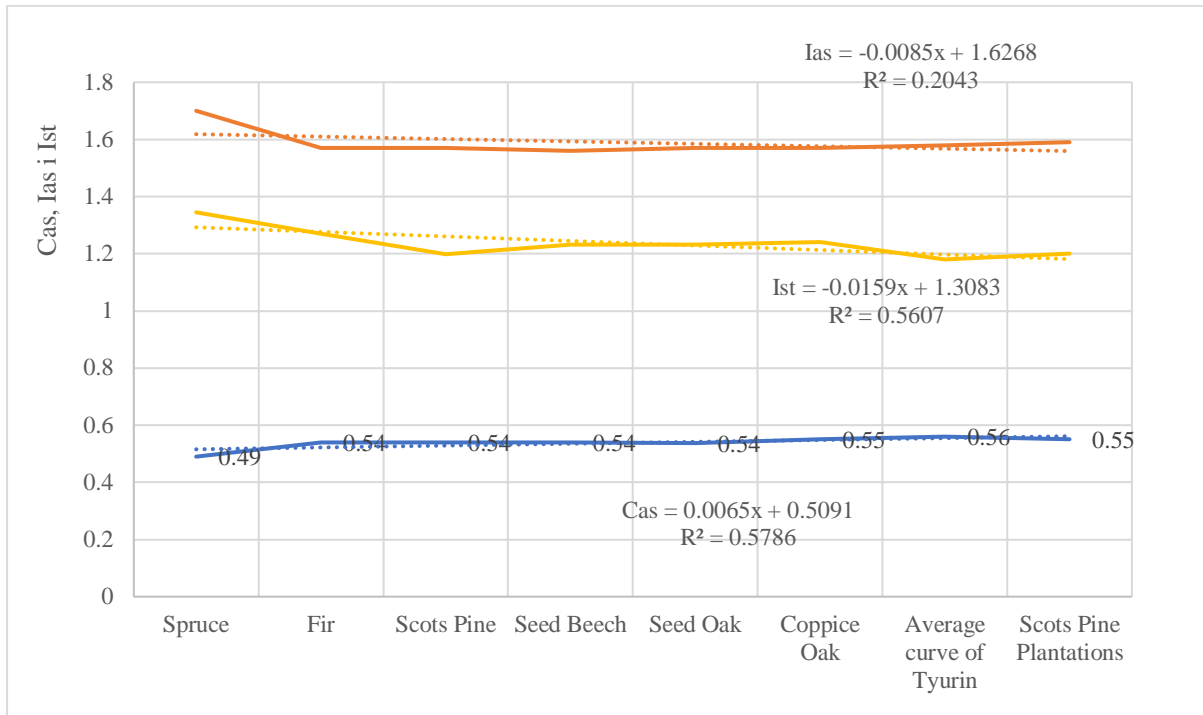


Figure 7. Coefficients (C_{as}) and indices of asymmetry (I_{as}), and indices of steepness (I_{st}) by tree species

From Fig. 7 it can be seen that the asymmetry coefficients for all tree species increase slightly, i.e. the asymmetry from spruce through the other species up to the

Conclusions

The study shows that the structural regularities of stands in terms of thickness (asymmetry of diameter distributions) are related to those in terms of height (steepness of height-diameter curves). For all studied species, distributions skewed to the right correspond to steeper height curves, which indicates higher

Average Tyurin Curve shifts to the left, and this is accompanied by a simultaneous decrease in the asymmetry indices and in the steepness indices.

productivity and better potential for wood volume. The newly applied indices – I_{as} and I_{st} – allow for an accessible and effective comparison of different types of stand structure of different tree species. The approach offers opportunities for a more accurate assessment of forest resources, and for improved decision-making in the field of silviculture.

References

1. Bronisz K., Mehtätalo L. Mixed-effects generalized height–diameter model for young silver birch stands on post-agricultural lands. *Forest Ecology and Management*. 2020; 460: 117901. – DOI: <https://doi.org/10.1016/j.foreco.2020.117901>.
2. Ciceu A., García-Duro J., Seceleanu I., Badea O. A generalized nonlinear mixed-effects height–diameter model for Norway spruce in mixed uneven-aged stands. *Forest Ecology and Management*. 2020; 477: 118507. – DOI: <https://doi.org/10.1016/j.foreco.2020.118507>.
3. Ciceu A., Pitar D., Badea O. Modeling the diameter distribution of mixed uneven-aged stands in the South Western Carpathians in Romania. *Forests*. 2021; 12(7): 958. – DOI: <https://doi.org/10.3390/f12070958>.
4. Ciceu A., Chakraborty D., Ledermann T. Examining the transferability of height–diameter model calibration strategies across studies. *Forestry: An International Journal of Forest Research*. 2023; Advance online publication. – DOI: <https://doi.org/10.1016/j.foreco.2021.119574>.
5. Da Silva B.G., Demétrio C.G.B., Sermarini R.A., Molenberghs G., Verbeke G., Behling A., Figura M.A. Height–diameter models: A comprehensive review with new insights on relationships to generalized linear models and differential equations. *International Forestry Review*. 2024; 26(4): 398–419. – DOI: <https://doi.org/10.1505/146554824839334687>.

6. Han Y., Lei Z., Ciceu A., Zhou Y., Zhou F., Yu D. Determining an accurate and cost-effective individual height–diameter model for Mongolian pine on sandy land. *Forests*. 2021; 12(9): 1144. – DOI: <https://doi.org/10.3390/f12091144>.
7. Lee D., Siipilehto J., Miina J., Niemistö P., Haapanen M., Hynynen J., Huuskonen S. Site index and stand characteristic models for silver birch plantations in southern and central Finland. *Forest Ecology and Management*. 2024; 563: 121998. – DOI: <https://doi.org/10.1016/j.foreco.2024.121998>.
8. Lebedev A.V. New generalized height–diameter models for birch stands in European Russia. *Baltic Forestry*. 2020; 26(2): 265–271. – URL: https://balticforestry.lammc.lt/bf/PDF_Articles/2020-26%5B2%5D/Baltic%20Forestry%202020.2_265-271.pdf.
9. Lebedev A.V., Kuzmichev V.V. Verification of two- and three-parameter simple height–diameter models for birch in the European part of Russia. *Journal of Forest Science*. 2020; 66(9): 375–382. – DOI: <https://doi.org/10.17221/76/2020-JFS>.
10. Petrin R., Markov I., Mihov I. Comparative studies on the structure of natural mature and middle-aged beech dendrocoenoses by height. *Management and Sustainable Development*. 2014; 16(49): 76-81. – URL: https://jmsd.bg/files/articles/49/49-10_R_Petrin_Markoff_Mihov.pdf (In Bulgarian).
11. Petrin R. Height structure of Scots pine stands. *Forestry ideas*. 2015; 21(2): 241-249. – URL: https://forestry-ideas.info/issues/issues_Index.php?journalFilter=69.
12. Petrin R., Markov I. Structure of spruce and fir dendrocoenoses by diameter. *Journal of Management and Sustainable Development*. 2015; 17(55): 97-102. – URL: https://jmsd.bg/files/articles/55/55-14_R_Petrin_I_Markoff.pdf (In Bulgarian).
13. Petrin R. Thickness structure of (coppice) tendril oak, winter oak and cerris oak dendrocoenoses. *Management & Sustainable development*. 2016; 18(57): 91-95. – URL: https://jmsd.bg/files/articles/57/57-16_R_Petrin.pdf.
14. Petrin R., Markoff I. Height structure of Coppice oak (Hungarian, Sessile and Turkey) stands. *Forestry ideas*. 2016; 22(2): 206-213. – URL: https://forestry-ideas.info/issues/issues_Index.php?pageNum_rsIssue=1&totalRows_rsIssue=10&journalFilter=56.
15. Petrin R. General regularities in the Height structures of Norway-spruce, Silver-fir and other dendrocoenoses. *Management & Sustainable development*. 2018; 20(69): 62-67. – URL: https://jmsd.bg/files/articles/69/69-12_R_Petrin.pdf.
16. Petrin R., Markoff I. Thickness structures of Natural and Artificial Scots pine dendrocoenoses. *Scientific researches of the Union of Scientists in Bulgaria-Plovdiv, series B. Natural Sciences and the Humanities*. 2018; XVIII: 51-58. – URL: <https://cyberleninka.ru/article/n/thickness-structures-of-naturaland-artificial-scots-pine-dendrocoenoses/viewer>.
17. Petrin R. General regularities of height-diameter curves in Hungarian oak, sesil oak and Turkey-oak high forests. *Silva Balkanica*. 2020; 21(2): 71-82. – URL: <https://archive.org/details/generalregulari21petr>
18. Petrin R. Index method and its application in studying forest stand structure and growth. *Scientific Works of the Union of Scientists in Bulgaria – Plovdiv, Series B: Natural Sciences and Humanities*. 2021; XXI: 113–125. – URL: https://usb-plovdiv.org/wp-content/uploads/2021/05/2021_estestveni_i_humanitarni_nauki_tom_XXI.pdf.
19. Petrin R. Growth-rate indices and growth type coefficients in height of broadleaved forest stands. *Forestry Engineering Journal*. 2022; 12(4): 47–59. – URL: <https://www.researchgate.net/publication/366910732> (Available via ResearchGate).
20. Petrin R. Simple asymmetry indicators tested in natural even-aged Scots pine (*Pinus sylvestris* L.) stands in Bulgaria. *Forestry Engineering Journal*. 2023; 13(3): 41–55. – DOI: <https://doi.org/10.34220/issn.2222-7962/2023.3/4>.
21. Petrin R. Comparative analysis of structure by thickness and height of oak forest dendrocoenoses using new indicators in studying asymmetry. *Scientific Works of the Union of Scientists in Bulgaria – Plovdiv, Series B: Natural Sciences and Humanities*. 2023a; XXI: 126–136. – URL: https://usb-plovdiv.org/wp-content/uploads/2023/06/2023_natural_sciences_and_humanities_vol_XXIV.pdf.

22. Petrin R. Oak dendrocoenoses' thickness structures determined by means of new indicators and comparing these with the respective height structures. *Nauka za gorata (Forest Science), Special Issue.* 2024; 199–207. – URL: https://naukazagorata.com/wp-content/uploads/2024/07/21_Petrin_OK.pdf.
23. Petrin R.D. Steepness and types of steepness of structure curves according to volume of aggregates of forest stands of different categories and tree species. *Silva Balcanica.* 2024a; 25(3): 33–43. – DOI: <https://doi.org/10.3897/silvabalcanica.25.e129570>.
24. Petrin R. Height structure and productivity of forest stands of different tree species. In: Proceedings of the 33rd International Scientific Conference "Management and Quality. Ecology and Management of Forest Ecosystems", October 9–10, 2024, Sofia. 2024; p. 132–140. – URL: <https://sskb.org/dokladi/2024/%D0%A1%D0%B1%D0%BE%D1%80%D0%BD%D0%B8%D0%BA%20%D0%9C%D0%9D%D0%9A%20%D0%95%D0%BA%D0%BE%D0%BB%D0%BE%D0%B3%D0%B8%D1%8F2024.pdf>.
25. Nuutinen Y., Mehtätalo L. Predicting height–diameter relationship in uneven-aged stands in Finland. *Forest Ecology and Management.* 2023; 549: 121486. – DOI: <https://doi.org/10.1016/j.foreco.2023.121486>.

Информация об авторе

✉ *Румен Димитров Петрин* – кандидат с.-х. (доктор) наук (по болгарской номенклатуре: образовательная и научная степень «доктор»), доцент, Научно-исследовательский институт леса Болгарской академии наук, бул. Климента Охридского, 132, г. София, 1756, Болгария. ORCID: <http://orcid.org/0000-0002-5831-5983>, email: lesni4eja2014@gmail.com.

Information about the author

✉ *Rumen D. Petrin* – Dr. Sci. (Forestry), Associated Professor, Forest Research Institute in Bulgarian Academy of Sciences, 132, St. Kliment Ohridski Blvd., Sofia, 1756, Bulgaria. ORCID: <http://orcid.org/0000-0002-5831-5983>, e-mail: lesni4eja2014@gmail.com.

✉ Для контактов/Corresponding author